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Modeling of capacitive MEMS microphone with square membrane or plate using integral method

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Abstract

An analytical approach describing the acoustic pressure field inside a thin layer of thermo-viscous fluid trapped between the square membrane or plate and planar backing electrode of the electrostatic receiver, which involves integral formulation, is presented. The use of integral formulation enables to choose between a model of membrane and plate easily and to avoid the limitations of the classical method whereby the acoustic field is expressed as a sum of eigenmodes coupled with modal expression of the diaphragm displacement. The analytically calculated pressure sensitivity of the receiver is compared to the numerically calculated one and the discrepancies are discussed.

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Keywords: Thin-film damping; MEMS microphones; thermoviscous fluid; pressure sensitivity; integral method

1. Introduction

The large variety of realizations of MEMS microphones [1] involves the need of precise theoretical models of such devices. Beyond the models of classical condenser microphones [2, 3], the works focused on miniaturized receivers [4, 5, 6] can be found in the literature. Owing to the accuracy sought and the measurement conditions that rise for metrological purposes today (e.g. acoustic gas thermometry [7]), the development of new analytical models useful in designing of new types of receivers and exploring their behavior at unusual ambient conditions (large static pressure and temperature range) are of interest.

The square geometry of the MEMS microphone considered herein is simple and thus advantageous from the point of view of microfabrication. It consists of a thin fluid gap of thickness h_g between a square diaphragm (membrane or plate) and a planar backing electrode, both having the same halfside a, and a peripheral cavity of volume V_c (see Figure 1). In the literature the model of such devices with membrane [4] or plate [5] is based on a solution for the acoustic field in the fluid film expressed as expansion of eigenfunctions, subjected to interface conditions or Neumann boundary conditions at the periphery of the fluid gap, coupled with the diaphragm displacement expressed usually in terms of eigenfunctions satisfying Dirichlet boundary conditions, which can lead to procedural difficulties. The aim of the presentation is to express the pressure field in the fluid layer in using the integral formulation with appropriate Green's function, which is not expressed as sum over eigenfunctions. Such a model has

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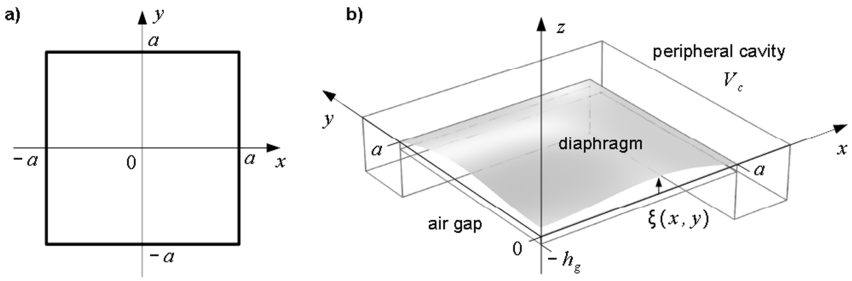

Figure 1: Geometry of the system: a) the dimensions of the square diaphragm and b) geometry of the transducer in the 1st quadrant.

been proposed recently for the transducer with square membrane [8]. An extension to the modeling of the transducer with square plate, which is the realistic approach in modeling of MEMS devices, is presented herein.

2. Analytical solution [8]

The propagation of the acoustic pressure in the air gap is governed by the wave equation (the time dependence being $e^{j\omega t}$)

$$\left(\partial^2_{xx} + \partial^2_{yy} + \chi^2\right) p(x,y) = -U(x,y), \tag{1}$$

where the source term $U(x,y) = \rho_0 \omega^2 \xi(x,y) / \left(h_g F_v\right)$ depends on the displacement of the diaphragm, $\xi(x,y)$, F_v is the mean value of the normalized profile function of the particle velocity across the air gap and the complex wavenumber χ accounts for the angular frequency ω of the field and the properties of the fluid, namely the compressibility and the density ρ_0 through the adiabatic speed of sound c_0, the heat capacity at constant pressure per unit of mass C_P, the specific heat ratio γ,the shear viscosity coefficient μ, and the thermal conduction coefficient λ_h (see for example [4, 6, 9]). The peripheral cavity is described by its impedance Z_c and the acoustic pressure inside this cavity p_c is supposed to be uniform. The normal derivative of the acoustic pressure at the periphery of the air gap $\partial_n p = -j\omega\rho_0 p_c / \left(4F_v Z_c h_g a\right)$ is calculated from the usual expression for the mean particle velocity in the air gap [9].

Since the system is symmetric, the acoustic pressure can be calculated only in 1st quadrant (for x>0, y>0) and the Green's function $G(x,x_0;y,y_0)$ appropriate for the solution of Eq. (1) is formed from the usual 2D Green's function $g(x,x_0;y,y_0) = -jH_0^-(\chi|\vec{r}-\vec{r}_0|)/4$ as follows

$$G(x,x_0;y,y_0) = g(x,x_0;y,y_0) + g(x,-x_0;y,y_0) + g(x,x_0;y,-y_0) + g(x,-x_0;y,-y_0), \tag{2}$$

where $H_n^-(z)$ denotes the Hankel function of the second kind of order n and $|\vec{r}-\vec{r}_0| = \sqrt{(x-x_0)^2+(y-y_0)^2}$. The integral formulation for the acoustic pressure inside the domain then takes the form

$$p(x,y) = \iint\limits_{(0,a)\times(0,a)} G(x,x_0;y,y_0)U(x_0,y_0)\mathrm{d}x_0\mathrm{d}y_0 + p_c I_G(x,y), \tag{3}$$

with $I_G(x,y) = j\omega\rho_0\left[\int_0^a G(x,x_0;y,a)\,\mathrm{d}x_0 + \int_0^a G(x,a;y,y_0)\,\mathrm{d}y_0\right] / \left(8F_v Z_c h_g a\right) + \int_0^a \partial_{y0} G(x,x_0;y,a)\,\mathrm{d}x_0 + \int_0^a \partial_{x0} G(x,a;y,y_0)\,\mathrm{d}y_0$. The mean value of the acoustic pressure at one of the air gap periphery gives approximately the value of the pressure inside the peripheral cavity $p_c = \langle p(x,a)\rangle_x$ (here for example at $y=a$, the symbol $\langle\cdot\rangle_x$ denotes the mean value over the x-coordinate).

The displacement of the diaphragm, driven by an incident acoustic pressure p_{inc}, is searched for in the form of expansion on Dirichlet set of orthogonal functions $\psi_{mn}(x,y)$ with associated eigenvalues k_{mn}, $\xi(x,y) = \sum_m \sum_n \xi_{mn}\psi_{mn}(x,y)$. For the square membrane (which is presented in this section, the plate is introduced in section 3) the modal coefficients ξ_{mn} are given by

$$\xi_{mn} = \frac{1}{T\left(k_{mn}^2 - K_M^2\right)} \iint\limits_{(-a,a)\times(-a,a)} \psi_{mn}(x,y)\left[p(x,y) - p_{inc}\right]\mathrm{d}x\,\mathrm{d}y, \tag{4}$$

where $K_M^2 = \omega^2 M_S / T$, T being the tension of the membrane and $M_S = h_d \rho_d$ the mass per unit area, h_d and ρ_d are the thickness of the diaphragm and its density, respectively. Introducing the acoustic pressure in the air gap (Eq. 3) into Eq. (4) gives the set of equations for the modal coefficients which, in the matrix form, can be expressed as

$$(-[\mathrm{A}]+[\mathrm{B}])(\Xi) = (\mathrm{C}), \tag{5}$$

where (Ξ) and (C) are, respectively, the column vectors of elements ξ_{mn} and

$$c_{mn} = -p_{inc} \iint\limits_{(-a,a)\times(-a,a)} \psi_{mn}(x,y)\,\mathrm{d}x\,\mathrm{d}y, \tag{6}$$

[B] is the diagonal matrix of elements $T(k_{mn}^2 - K_M^2)$, and where the matrix [A] of elements $A_{(\mathrm{mn}),(\mathrm{qr})}$ is given by

$$\begin{aligned} A_{(\mathrm{mn}),(\mathrm{qr})} = \frac{\rho_0 \omega^2}{h_g F_v} \int_{-a}^{a}\int_{-a}^{a} \Bigg[& \psi_{mn}(x,y) \int_0^a\int_0^a G(x,x_0;y,y_0)\ \psi_{qr}(x_0,y_0)\,\mathrm{d}x_0\mathrm{d}y_0 - \frac{1}{1+\langle I_G(x,a)\rangle_x} \\ & \cdot \int_0^a\int_0^a \langle G(x,x_0;a,y_0)\rangle_x\ \psi_{qr}(x_0,y_0)\,\mathrm{d}x_0\mathrm{d}y_0 \Bigg\{ \left[\frac{j\omega\rho_0}{8F_v Z_c h_g a}\psi_{mn}(x,y) + \partial_y \psi_{mn}(x,y) \right] \\ & \cdot \int_0^a G(x,x_0;y,a)\,\mathrm{d}x_0 + \left[\frac{j\omega\rho_0}{8F_v Z_c h_g a}\psi_{mn}(x,y) + \partial_x \psi_{mn}(x,y) \right] \int_0^a G(x,a;y,y_0)\,\mathrm{d}y_0 \Bigg\} \Bigg] \mathrm{d}x\,\mathrm{d}y. \end{aligned} \tag{7}$$

3. Model of the transducers with square membrane and plate

The well-known set of orthogonal symmetrical eigenmodes $\psi_{mn}(x,y) = \cos(k_{x_m} x)\cos(k_{y_m} y)/a$ [4] is used for the square membrane, the elements of the column vector (C) (Eq. 6) are then expressed as $c_{mn} = -p_{inc}\, 16a(-1)^{\frac{m-1}{2}}(-1)^{\frac{n-1}{2}} / (mn\pi^2)$.

In the case of square plate clamped at its periphery the set of orthogonal symmetrical eigenmodes is given as follows [5]

$$\psi_{mn}(x,y) = \frac{1}{2a}\left[\frac{\cos(k_{x_m} x)}{\cos(k_{x_m}\,\mathrm{a})} - \frac{\cosh(k_{x_m} x)}{\cosh(k_{x_m}\,\mathrm{a})}\right]\left[\frac{\cos(k_{y_n} y)}{\cos(k_{y_n}\,\mathrm{a})} - \frac{\cosh(k_{y_n} y)}{\cosh(k_{y_n}\,\mathrm{a})}\right], \tag{8}$$

with $k_{x_m} = z(m)/a$, $k_{y_n} = z(n)/a$, $z(i)$ being the *i*-th solution of the equation $\tan(z) = -\tanh(z)$. In this case [B] is the diagonal matrix of elements $D\left((k_{x_m}^2 + k_{y_n}^2)^2 - K_P^2\right)$ with $K_P^2 = \omega^2 M_S / D$, $D = Eh_d / \left[12\left(1-\upsilon^2\right)\right]$ is the flexural rigidity of the plate, E and υ being the Young's modulus and the Poisson's ratio, respectively. The elements of the column vector (C) (Eq. 6) are given by $c_{mn} = -p_{inc} \dfrac{8a}{z(m)z(n)} \tan[z(m)]\tan[z(n)]$.

Table 1: Geometrical and mechanical parameters of the transducer.

Parameter	Value	Unit
Diaphragm half-side a	0.5×10^{-3}	m
Fluid gap thickness h_g	10×10^{-6}	m
Diaphragm thickness h_d	10×10^{-6}	m
Cavity volume V_c	10^{-10}	m^3
Silicon diaphragm density ρ_d	2330	kg/m^3
Membrane tension T	945	N/m
Plate Young's modulus E	160×10^9	Pa
Plate Poisson's ratio v	0.27	-

The pressure sensitivities of the miniaturized acoustic receivers $\sigma = -U_0\bar{\xi}/\left(p_{inc}h_g\right)$ where $\bar{\xi}$ is the mean diaphragm displacement and U_0 is the polarization voltage (herein $U_0 = 30\text{V}$) with dimensions and other properties given in Table 1 (the typical properties of air can be found for example in [6]), calculated using both this integral method and a finite element method [10, 11] for the transducer with membrane and plate, are shown in Figure 2. A good agreement between the analytical and numerical results is obtained for the transducer with square membrane (up to 600 kHz). In case of the square plate the analytically calculated sensitivity at low frequencies (below the first eigenmode) is approximately 3 dB lower than the numerically calculated one and the damping of the system seems to be underestimated. This can be explained by the fact that the eigenfunctions used (Eq. 8) are approximated as mentioned in reference [5]. Note that for a 1D plate (rectangular beam) the eigenfunctions based on Eq. (8) are exact and the method presented herein is particularly well suited for this case.

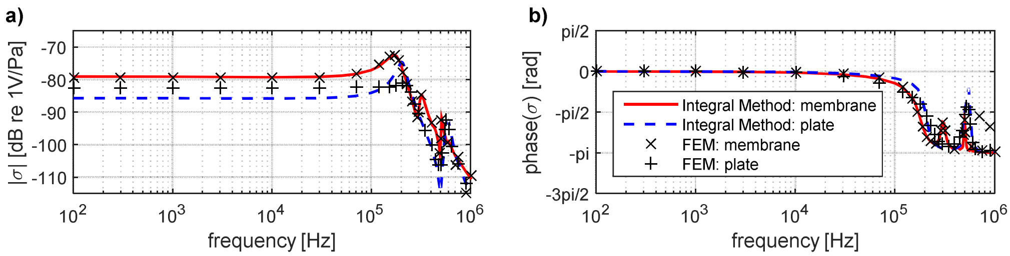

Figure 2: a) Magnitude and b) phase of the pressure sensitivity of the miniaturized receiver calculated analytically for transducer with membrane (full red line) and with plate (dashed blue line) and calculated numerically (”×” marks for membrane and ”+” marks for plate).

4. Conclusions

When searching for precise analytical models of miniaturized microphones where the thermal and viscous boundary layers effect and coupling effects are taken into account, the integral formulation (along with the Green’s function which is not expressed as a sum over eigenfunctions) presented herein seems to be well suited for this aim. A good agreement between the analytically calculated pressure sensitivity of the receiver and the numerically calculated one (used as a reference against which the analytical results are tested) is obtained in the case of miniaturized microphone with square membrane while the model of the microphone with square plate seems to slightly underestimate the sensitivity and the damping in the system due to the approximated eigenfunctions, but still remains interesting for practical applications. The validity of this analytical model for describing the behaviour of condenser microphones should be experimentally checked in various measurement conditions of interest for metrological purpose.

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